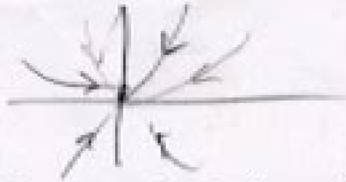
if $\alpha > 4\omega_0^2$ two real solutions, both <0



(b) draw a phase-plane (that is u' vs. u).

(c) interpret the motion of the mass and relate to what's going on in the phase plane diagram.

(d) find the general solution using the methods of Chapter 3.

(e) convince yourself that the two general solutions you've derived in parts (a) and (d) represent the same solution.

(f) do (a)-(e) for the equation $y'' + 3y' + 2y = 0$.

7. Given the follow differential equation,

$$\begin{aligned} y''' + p(t)y'' + q(t)y' + r(t)y &= f(t), \\ y(0) &= \alpha, \\ y'(0) &= \beta, \\ y''(0) &= \gamma, \end{aligned}$$

(a) write it as a system of first order differential equations *in matrix form*. That is, in the form $\vec{x}' = A\vec{x} + \vec{b}$ with $\vec{x}(0) = \vec{x}_0$ where you tell me how the quantities $\vec{x}$, A, $\vec{b}$, and $\vec{x}_0$ are defined! Assume that $p(t)$, $q(t)$, $r(t)$ and $f(t)$ are given functions and α, β and γ are given constants.

(b) Assume three solutions $y_1(t)$, $y_2(t)$ and $y_3(t)$ of the homogeneous equation are known. By using the matrix formulation of point (a), find an expression for a particular solution by using the variation of parameters method (no need to carry out matrix multiplications or inversions explicitly, just write down an expression involving fundamental matrices). Would *any* three solutions of the homogeneous equation produce a particular solution? Apply what you found to the example

$$p(t) = -1, \quad q(t) = 1, \quad r(t) = -1,$$

given that three solutions are (check it!)

$$y_1(t) = e^t, \quad y_2(t) = \cos(t), \quad y_3(t) = 2e^t + 3\cos(t).$$

8. Which of the following homogeneous systems have nontrivial solutions? You do not necessarily need to *find* the solution. State your reasons.

(a)

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$