"Fish gotta swim, birds gotta fly:" the mathematics and mechanics of moving things

The scientific method is arguably the single most important achievement of the modern era. Together with its technological implications, it has shaped the world in the last four centuries, both physically and culturally, like no other element in the history of mankind. It is the mission of research universities to teach this method and how to implement it.

Of the three steps of the method, experimental observation, its interpretation, and the testing of the predictive power that this interpretation provides versus new experimental evidence, Mathematics, and especially Applied Mathematics, is essential at the interpretation and predictive stage. It is somewhat unfortunate that the mathematical skills necessary for forefront research are in general well beyond the reach of the average freshman, so that this component of one's education has to be deferred until much later. In fact, this experience is nowadays generally missing altogether from the undergraduate experience. Perhaps even more unfortunate is the fact these mathematical skills have to be acquired in an exercise-solution environment where little of the motivation and none of the exhilaration of predicting the outcome of a process can be conveyed. With this in mind, we propose to offer a freshman seminar program where students will have hands-on exposure to a class of physical and computer experiments and will be stimulated to provide and consequently test the mathematical interpretation of their observations.

To be successful, experiments have to be drawn from a class where even modest mathematical knowledge (at most at calculus level) can provide a starting point for a working mathematical model. Perhaps even more critically, these experiments need to be able to surprise and challenge some elements of the mental picture of the world which we collectively refer to as "common sense." For this task, we propose to use the fluid mechanics lab that our applied math group is in the course of developing, together with the computational environment of the Mathematics Undergraduate Laboratory. The set of experiments will consist of simple setups from three areas:

Mechanics

Students will observe classical examples of conservation of momentum and angular momentum. They will be encouraged to give their own formulation of these laws, and test these conservation laws on simple devices, like a rotating table and a swing. They will be asked to contrast this knowledge with the observation of how a deformable body (like in the classical example of a cat always landing on its feet) can manage to rotate itself during free fall. They will be encouraged to observe how a book tossed in the air can never be made to stabilize its rotation around its middle axis. Concepts like stability and instability will thus be introduced and their mathematical definition visualized. The students will then be exposed to possible applications of these concepts to design efficient mechanisms for changing the orientation of artificial satellites in space and asked to provide simple estimates of efficiency. Computer simulations of several experiments will complement the physical observations and demonstrate the effectiveness of mathematical modelling.

Fluid Dynamics

One of the intriguing aspects of fluid dynamics is that incredibly complex fluid flows can nevertheless be classified according to a single nondimensional number, the so-called

Reynolds number. This number provides an estimate of the relative intensity of the fluid's inertial versus frictional forces, the viscosity of the fluid. The fluid dynamics human beings are mostly exposed to occurs at regimes where the fluid inertia dominates over viscosity, i.e., for large Reynolds numbers. Such is the case, for instance, with the flight of birds or the swimming of fish, or with our own flying and swimming devices. A strange and unfamiliar world, in many ways more essential to life than the one we are familiar with, exists in the opposite regime of small Reynolds numbers. This is the world experienced by small living creatures from micro-organisms to insects. Quite fortunately, small Reynolds numbers regimes can be realized in a relatively low tech lab environment, simply by using highly viscous fluids like glycerin and corn syrup. In this lab, the students will perform a set of experiments that will challenge their intuition based on a lifetime of observing large Reynolds number flows (such as the reversibility of small Reynolds flows) and bring forth the essential features of how locomotion in fluids can ultimately be achieved by completely different approaches. By observation, the students will "discover" the simple scaling laws that govern such experiments. They will test these laws in subsequent visits to the fluid lab and by computer simulations that will be made available to them.

Iterated Maps and Chaotic Flows

Most freshmen have been exposed to the concept of a function as an assigned rule that associates a number to another. Most of them might even be able to graph such a rule. Very few of them, however, have been trained to look at what happens when this rule is applied over and over again in an iterative fashion. This concept is basic to many branches of mathematics, and is of fundamental importance in scientific computing. Remarkably, functional iteration can be effectively visualized on even the simplest computers. Many of the beautiful plots generated this way, the so-called fractals or Julia sets, have become commonplace nowadays, however seldom their connection with the real world is established. In this part of the seminar, the students will learn how to associate the concept of iterated functions with the practical problem of finding the roots of quadratic and more general polynomial equations. They will then be exposed to the concept of stability of these roots under iteration. Finally, these concepts will be connected to some of the findings in the other two parts of the seminar series, where simple effects of instabilities, like in the case of rigid body rotation or in the question of the reversibility of low Reynolds number flows, have been observed.

The seminar will encourage verbal and written communication, and the experience of team work and interactions will constitute an essential and highly beneficial part of the learning experience.