The predictability of chance and its applications in applied mathematics.

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Suggested Reading: *The Pleasures of Probability*, **Richard Isaac, Springer**

The notions of randomness and chance permeate our everyday life. Familiar expressions of speech such as "What are the odds that . . .", or "the chances are a million to one that . . .", are common place, yet it is fair to say that the concepts of chance and predictability are poorly understood by most. As evidence, consider the popular "Monty Hall paradox:" A participant in a prize extraction game is given two choices out of three available options. One of the three options contains the grand prize. After selecting two, the participant is shown the outcome of one of the choices (obviously not the one corresponding to the grand prize if it were to be one of them), and then asked whether he or she would like to switch the previous choice. Elementary considerations show that the best strategy in this game is to accept the switch, something that most people find so surprising that explanations are almost invariably met with skepticism. Although understanding these problems requires nothing more than elementary probability, it still goes against most people's "common sense." The fact that this can be challenged with simple classroom and computer demonstrations makes this area ideally suited to the guiding principle of this seminar in Applied Mathematics.

This first year seminar is organized as follows. Starting with simple brain teasers like the example above, we will develop an understanding of basic probability theory. We will outline the content of the central limit theorem and explore its ramifications through simple classroom experiments involving random events. In addition, we will introduce simple numerical algorithms involving randomness to complement in-class experiments. These will also provide a cartoon of more sophisticated algorithms used in the field of scientific computation. We will then be able to compare the outcomes of numerical runs with those coming from numerical integration of simple dynamical systems exhibiting chaotic behavior, and address the issue of true randomness versus deterministic chaos by looking at systems that exhibit synchronization. In all cases, heavy emphasis will be placed on "discovering" the rules by classroom or computer experimentation which will then be backed up by an introduction to the theory.

The central limit theorem lies at the heart of many of these examples. We will discover that the normal (Gaussian) distribution appears naturally when summing suitably scaled independent, identically distributed random variables drawn from more or less arbitrary probability distributions (under suitable assumptions). As an example, we will build histograms detailing the number of heads obtained from flipping N coins. Clearly, when N=1, the histograms will contain only two possible states with nearly equal weight, but as N increases, the familiar bell-shaped curve appears. Through classroom and computer exercises, students will discover that the relative width (standard deviation) of these bell curves decreases as the number of trials increases (for a suitably scaled sum).

Building on these basic ideas, we will begin to explore the subject of Monte Carlo methods in scientific computing. For instance, we will discover how π can be accurately computed simply by throwing darts at a circle inscribed in a square. In actuality this will be done through computer experiments using random number generators. We will derive the appropriate error bounds through experimentation which will then be confirmed through the central limit theorem. In another application, the basic feature of the heat propagation equation will be illustrated, and the students will be able to simulate it by using computer-generated random walks. Additionally, concepts fundamental to mathematical physics and quantum chemistry (such as path integral representations) may be introduced through such elementary examples and will help to introduce and familiarize students with advanced techniques used in current research in physics, chemistry, and applied mathematics.

We will learn what it means for a number to be truly random by attempting to build our own random number generators. Hints on how to do this through iterations of a simple rule will be provided, and computer exploration of various rules encouraged. A poor choice of rule will become manifest when the random number generator is used for the computation of π outlined above, and this will illuminate the essential difficulty that this subject entails. Random number generators lead naturally to the notion of iterative maps. This is a rich field, where concepts like chaotic dynamics, periodicity, and ergodicity can be easily illustrated.

The format of the seminar will be that of general lectures alternating with a sequence of guided discovery experiments, mainly through the existing Undergraduate Computing Laboratory. Towards the end of the course, teams will be formed and asked to choose among a set of topics (e.g., Monte Carlo methods for computing volumes, or the influence of bias in the pool of respondents to a poll) for a more in-depth investigation, to form the basis of a report to be completed by the end of the course.