

1.

$$y(x) = \frac{1}{Ce^{-x} - e^{-x} \int^x se^s ds} = \frac{1}{1 - x + Ce^{-x}}$$

2. See solution of logistic equation worked out in class. R is the limit if $R > 0$, 0 if $R < 0$.

3. (a)

$$t = 300 \log 10$$

(b)

$$t = 300 \log \left(\frac{47}{2} \right)$$

4. Equilibria are 0 and $a^{\frac{1}{3}}$. If $k > 0$, $a^{\frac{1}{3}}$ is stable, 0 unstable; if $k < 0$ viceversa, provided in both cases $a > 0$.

5.

$$y(x) = Ce^{\frac{1}{4}x^4} + e^{\frac{1}{4}x^4} \int^x (1 - s^3)e^{(s-\frac{1}{4}s^4)} ds = Ce^{\frac{1}{4}x^4} + e^x$$

6.

$$y(t) = 1 \pm \sqrt{C - \frac{2t^3}{3}}$$

interval of definition of solution determined by positivity of square root.

7. (a)

$$P(6\text{yrs}) = \frac{e^{12} - 1}{2} \simeq 81,377$$

(b)

$$h \geq 2P(6\text{yrs})$$

(c) Slightly longer than a year.