Optional HW A:

17.11.4: $f(x) = 50(1 - H(x-4))$

$$\hat{f}_c(\omega) = \frac{2}{\pi}\int_0^\infty f(x)\cos\omega x\,dx$$

$$= \frac{2}{\pi}\cdot 50\cdot\int_0^4 \cos\omega x\,dx$$

$$= \frac{100}{\pi}\cdot\frac{1}{\omega}\sin\omega x\Big|_0^4$$

$$= \frac{100}{\pi}\,\frac{\sin 4\omega}{\omega}$$

$$\hat{f}_s(\omega) = \frac{2}{\pi}\int_0^\infty f(x)\sin\omega x\,dx$$

$$= \frac{100}{\pi}\int_0^4 \sin\omega x = \frac{-100}{\pi\omega}\cos\omega x\Big|_0^4$$

$$= -\frac{100}{\pi\omega}(\cos 4\omega + 1)$$

17.11.9:

a) $u'' - 9u = 50e^{-3x}$, $0 < x < \infty$, $u(0) = 0$, $u(\infty) < \infty$

$$\Rightarrow -\omega^2\hat{u}_s + \omega u(0) - 9\hat{u}_s = \frac{50\omega}{\omega^2+9}$$

$$\Rightarrow \hat{u}_s = -\frac{50\omega}{(\omega^2+9)^2} \Rightarrow u(x) = -\frac{50}{6}xe^{-3x}$$

17.11.9:

b) $u'' - 9u = 50e^{-3x}, \quad u'(0)=0, \quad u(\infty) < \infty$

$$\Rightarrow -\omega^2 \hat{u}_c - u'(0) - 9\hat{u}_c = \frac{3}{\omega^2+9} \cdot 50$$

$$\Rightarrow \hat{u}_c = -150 \frac{1}{(\omega^2+9)^2}$$

$$= -150 \frac{1}{\omega^2+9} \cdot \frac{1}{\omega^2+9}$$

$$\Rightarrow \quad u(x) = \frac{1}{2} \int_0^\infty \left(e^{-3|x-\xi|} + e^{-3(x+\xi)}\right) e^{-3\xi} \, d\xi$$

$$= \frac{1}{2} \int_0^\infty e^{-3(|x-\xi|+\xi)} d\xi + \frac{e^{-3x}}{2} \int_0^\infty e^{-6\xi} d\xi$$

$$= \frac{1}{2}\left(\int_0^x e^{-3x} d\xi + \int_x^\infty e^{3x} e^{-6\xi} d\xi\right) + \frac{e^{-3x}}{2} \left(-\frac{1}{6} e^{-6\xi}\Big|_0^\infty\right.$$

$$= \frac{1}{2} x e^{-3x} + \frac{1}{12} e^{-3x} + \frac{1}{2} e^{3x} \left(-\frac{1}{6} e^{-6\xi}\Big|_x^\infty\right.$$

$$= \frac{1}{2} x e^{-3x} + \frac{1}{12} e^{-3x} + \frac{1}{12} e^{-3x} = \frac{1}{2} \cancel{x} e^{-3x} + \frac{1}{6} e^{-3x}$$