Section 1.2

Homework 1 Solutions

(1)

6. The system is in rref already.

$$\left| \begin{array}{l} x_1 = 3 + 7x_2 - x_5 \\ x_3 = 2 + 2x_5 \\ x_4 = 1 - x_5 \end{array} \right|$$

Let $x_2 = t$ and $x_5 = r$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3+7t-r \\ t \\ 2+2r \\ 1-r \\ r \end{bmatrix}$$

29. Since the number of oxygen atoms remains constant, we must have $2a + b = 2c + 3d$.

Considering hydrogen and nitrogen as well, we obtain the system $\left| \begin{array}{rcl} 2a + b &=& 2c + 3d \\ 2b &=& c + d \\ a &=& c + d \end{array} \right|$

or

$$\left| \begin{array}{l} 2a + b - 2c - 3d = 0 \\ 2b - c - d = 0 \\ a - c - d = 0 \end{array} \right|, \text{ which reduces to } \left| \begin{array}{l} a - 2d = 0 \\ b - d = 0 \\ c - d = 0 \end{array} \right|.$$

The solutions are $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \\ t \end{bmatrix}$.

To get the smallest positive integers, we set $t = 1$:

$2NO_2 + H_2O \longrightarrow HNO_2 + HNO_3$

41. We know that $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{w}_1 + m_2\vec{w}_2$ or $m_1(\vec{v}_1 - \vec{w}_1) + m_2(\vec{v}_2 - \vec{w}_2) = \vec{0}$

or $\left| \begin{array}{l} -3m_1 + 2m_2 = 0 \\ -6m_1 + 4m_2 = 0 \\ -3m_1 + 2m_2 = 0 \end{array} \right|$.

We can conclude that $m_1 = \frac{2}{3}m_2$.

Section 1.3

3. This matrix has rank 1 since its rref is $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

4. The matrix is rank 2 since its rref is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

②

14. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ or $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} =$
$\begin{bmatrix} 1 \cdot (-1) + 2 \cdot 2 + 3 \cdot 1 \\ 2 \cdot (-1) + 3 \cdot 2 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

33. The ith component of $A\vec{x}$ is $[0\ 0\ \ldots\ 1\ \ldots\ 0] \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_i \\ \ldots \\ x_n \end{bmatrix} = x_i$. (The 1 is in the ith position.)

Therefore, $A\vec{x} = \vec{x}$.

35. Write $A = [\vec{v}_1\ \vec{v}_2\ \ldots\ \vec{v}_i\ \ldots\ \vec{v}_m]$, then

$$A\vec{e}_i = [\vec{v}_1\ \vec{v}_2\ \ldots\ \vec{v}_i\ \ldots\ \vec{v}_m] \begin{bmatrix} 0 \\ 0 \\ \ldots \\ 1 \\ \ldots \\ 0 \end{bmatrix} = 0\vec{v}_1 + 0\vec{v}_2 + \cdots + 1\vec{v}_i + \cdots + 0\vec{v}_m = \vec{v}_i = i\text{th column of } A.$$

Matlab: Enter rrefmovie() to see the answer.