Math 31 - Exam 1, Fall 2002 Solutions

Name: ______________________________________ (please print)

I pledge that I have neither given, nor received any unauthorized assistance on this exam.

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(signature)

1. Answer the following questions using Fig. 1 below.

a) What are the asymptotes of f(x)?

Horizontal asymptotes: y = 2 and y =0

6 points.

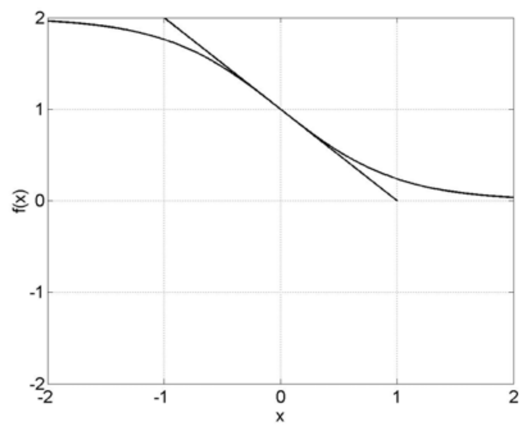


Fig. 1

b) Estimate the slope of f(x) at x = 0.

$$\text{slope} = \frac{0-2}{1-(-1)} = -1$$

6 points

c) In Fig. 2, sketch f′(x).

6 points.

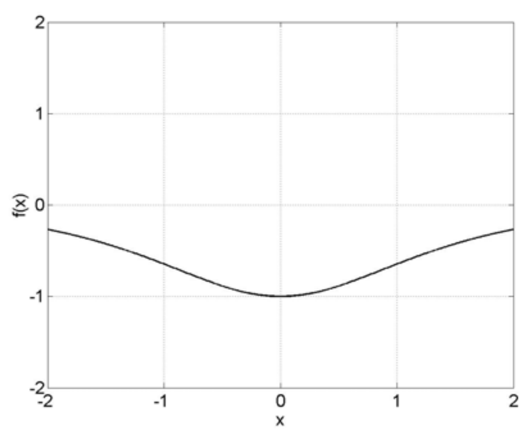


Fig. 2

2. Give the precise ε, δ definition of $\lim_{x\to 2}\left(x^2-5\right)=-1$.

Let ε be any positive number. Then there exists a δ > 0, such that
$0<\left|x-2\right|<\delta \Rightarrow \left|x^2-4\right|<\varepsilon$

6 points.

3. Evaluate the following limits or state that it does not exist.

a)

$$\lim_{x\to 3}\frac{x^2-8x+15}{3x-9}=\lim_{x\to 3}\frac{x^2-8x+15}{3\left(x-3\right)}=\lim_{x\to 3}\frac{\left(x-3\right)\left(x-5\right)}{3\left(x-3\right)}$$
$$=\lim_{x\to 3}\frac{\left(x-5\right)}{3}=\frac{-2}{3}$$

8 points.

b)

$$\lim_{\theta\to 0}\frac{\theta\cos\left(2\theta\right)}{\sin\left(5\theta\right)}=\lim_{\theta\to 0}\frac{5\theta\cos\left(2\theta\right)}{5\sin\left(5\theta\right)}=\frac{1}{5}\left(\lim_{\theta\to 0}\frac{5\theta}{\sin\left(5\theta\right)}\right)\left(\lim_{\theta\to 0}\cos\left(2\theta\right)\right)$$
$$=\frac{1}{5}$$

8 points.

c)

$$\lim_{x\to 3^-}\frac{x^2}{x^2-9}=-\infty \quad \text{limit does not exist}$$

8 points.

d)

$$\lim_{x\to\infty}\frac{5+6x-3x^3}{4x^3-5x}=\lim_{x\to\infty}\frac{\frac{5}{x^3}+\frac{6x}{x^3}-\frac{3x^3}{x^3}}{\frac{4x^3}{x^3}-\frac{5x}{x^3}}=\lim_{x\to\infty}\frac{\frac{5}{x^3}+\frac{6}{x^2}-3}{4-\frac{5}{x^2}}$$
$$=-\frac{3}{4}$$

8 points.

4. Use the limit definition of the derivative to find f′(x) if $f\left(x\right)=\frac{1}{x^2+4}$.

$$f'\left(x\right)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}=\lim_{h\to 0}\frac{\frac{1}{\left(x+h\right)^2+4}-\frac{1}{x^2+4}}{h}$$

$$=\lim_{h\to 0}\frac{x^2+4-\left[\left(x+h\right)^2+4\right]}{h\left[\left(x+h\right)^2+4\right]\left(x^2+4\right)}=\lim_{h\to 0}\frac{x^2+4-x^2-2xh-h^2-4}{h\left[\left(x+h\right)^2+4\right]\left(x^2+4\right)}$$

$$=\lim_{h\to 0}\frac{-2xh-h^2}{h\left[\left(x+h\right)^2+4\right]\left(x^2+4\right)}=\frac{-2x}{\left(x^2+4\right)^2}$$

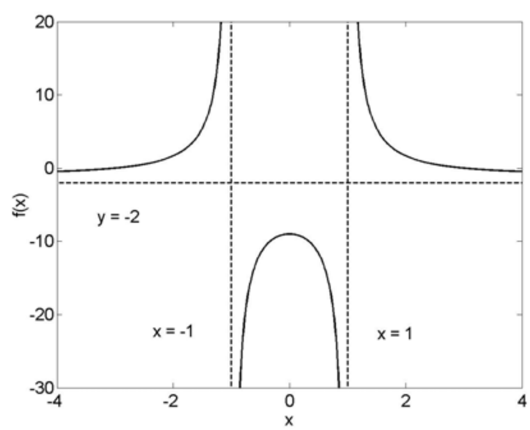
10 points.

1. Consider the function $f\left(x\right)=\frac{9-2x^2}{x^2-1}$. Find the horizontal and vertical asymptotes of f(x). Sketch the graph of f(x). Be sure to label the values of all horizontal and vertical asymptotes.

Horizontal asymptotes:

$$\lim_{x\to\infty}\frac{9-2x^2}{x^2-1}=\lim_{x\to\infty}\frac{\frac{9}{x^2}-2}{1-\frac{1}{x^2}}=-2$$

$$\lim_{x\to-\infty}\frac{9-2x^2}{x^2-1}=\lim_{x\to-\infty}\frac{\frac{9}{x^2}-2}{1-\frac{1}{x^2}}=-2$$



Vertical Asymptotes:

$f\left(1\right)=\frac{7}{0}$ and $f\left(-1\right)=\frac{7}{0}$. Therefore,

x = 1 and x = -1 are asymptotes.

16 points.

6. A particle's position is given by $f(t) = 4 - (t-2)^2$.

a) What is the average velocity over the interval [0,4]?

$$\text{Avg. vel.} = \frac{\text{distance traveled}}{\text{elapsed time}} = \frac{f(4)-(0)}{4-0}$$
$$= \frac{0-0}{4} = 0$$

6 points.

b) Compute the instantaneous velocity f′(t).

$$\text{Inst. vel.} = \lim_{h \to 0} \frac{4-(t+h-2)^2 - \left[4-(t-2)^2\right]}{h}$$
$$= \lim_{h \to 0} \frac{4-\left[(t-2)^2 + 2h(t-2) + h^2\right] - \left[4-(t-2)^2\right]}{h}$$
$$= \lim_{h \to 0} \frac{-2h(t-2)-h^2}{h} = -2t+4$$

6 points.

c) What is the instantaneous velocity at t = 4?

$$f'(4) = -2(4) + 4 = -4$$

6 points.